# A Hybrid Trajectory Planning Strategy for Intelligent Vehicles with Collision Avoidance

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**Abstract:** This paper presents a hybrid trajectory planning strategy with interpolation curve optimization method under the constraints of collision avoidance and vehicle dynamics. With an objective function consisting of a weighted sum of comfort and efficiency, the cubic polynomial curve is firstly optimized under the initial state and the terminal state constraints to generate a reference trajectory. Then, in order to generate collision-free trajectory, lateral and longitudinal turning factors are introduced to cluster the reference trajectory. According to the dimension relationship between vehicle and road, the maximum lane change distance and the maximum turning curvature, the number of trajectories in the cluster is constrained. Furthermore, an algorithm is designed to detect the collision of the trajectories. The priority of the trajectories is defined to determine the optimal trajectory. Finally, a lateral vehicle dynamic model is established. To judge the feasibility, under the condition of dynamic constraints, the reference trajectory is tracked with optimal control signal and judged by the tracking effect. In this way, the collision risk is eliminated, and the trajectory performance is improved. Simulation results show that the trajectory planning strategy is effective in different scenarios.

**Key Words:** Intelligent vehicles, trajectory planning, collision avoidance, vehicle dynamics

## 1. Introduction

Currently, the commonly used vehicle trajectory planning methods include graph search methods and parameter optimization methods [1]. Graph search methods use sampling trajectory primitives (arcs, helices, etc.) which are dynamically feasible to establish the graph, and then conducts sequential search, and generates the solution sequence of geometric primitives. The parametric optimization method is continuous optimization and can produce an analytical solution of a high order parameterized geometric trajectory. The graph search method generates global optimal solutions in discrete networks and is often used in regional trajectory planning, such as parking lots. Parametric optimization methods, which seek local optimal solutions in continuous solutions, are often used in structured trajectory planning.

Parameter optimization methods mainly have two branches: model prediction method and interpolation curve method [2].

Model predictive method uses the vehicle model to achieve optimal control. State variables that need to be constrained, such as speed and trajectory curvature, are used as control variables to obtain the predicted trajectory. Cost functions are defined according to the information of trajectory terminal deviation and danger indicators, and then the final control input is found through optimization to obtain the optimal executable trajectory under the optimal control signal. The model prediction method can fully consider the dynamic constraints of the vehicle and ensure the executability of the planned trajectory. The defect is that the complexity of the model and the efficiency of the algorithm cannot be guaranteed at the same time, and one of them must be sacrificed [3]. In addition, the model prediction method usually adopts numerical method to solve the curve parameters corresponding to the terminal state, and its convergence is often hard to be achieved [4].

Interpolation curve optimization inserts parameterized curves between existing target trajectory points, and establishes cost functions [1] from the perspectives of comfort and safety to optimize parameterized curves. Interpolation curve optimization in the reference line coordinate system can ensure that the planned trajectory adapts to the changes of road curvature [5]. The advantage of interpolation curve optimization method is that it is convenient to consider comfort, safety and other properties from the perspective of kinematics of the trajectories. The current problem of this method is that it is difficult to consider the influence of vehicle dynamics constraints, tire-road relationship and other factors in real-time planning.

This paper presents a hybrid trajectory planning strategy with interpolation curve optimization method under the constraints of collision avoidance and vehicle dynamics. The reference trajectory is tracked with optimal control signal and judged by the tracking effect. The collision risk is eliminated, and the trajectory performance is improved.

## 2. Reference Trajectory Generation

The first step of using interpolation curve optimization for intelligent vehicle trajectory planning is to determine the generation space of interpolation curve. At present, there are two kinds of coordinate systems in common use. Trajectory planning in the world coordinate system has been widely studied. The method with Frenét frame [6] has been applied in Baidu driverless open-source platform for intelligent vehicle trajectory planning, which can adapt the planned

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trajectory to follow the road curvature, but it is inconvenient to analyze vehicle dynamics constraints in Frenét frame.

In this paper, in the world coordinate system, suitable curve types are selected from the following interpolation curves for trajectory planning with fully considering vehicle dynamics constraints.

The curve types commonly used in trajectory planning include polynomial curve, spline curve, hyperbolic tangent curve, helix curve, sinusoidal curve, etc[1,7,11].

Helical trajectory can control the maximum curvature of the trajectory, but its lateral acceleration is not smooth. This problem also exists in sinusoidal trajectories. Spline trajectories can meet the continuity of multi-order derivatives and the spatial constraints of trajectory by adjusting control points, but the large number of parameters is not conducive to curve optimization and analysis of dynamic constraints [7]. Hyperbolic tangent curve has few trajectory parameters and all derivatives are smooth. However, due to its special curve shape, it is only applicable to special scenes such as lane change [8].

For polynomial trajectories, these problems can be reduced because the degree of the polynomial can be adjusted to achieve the desired performance and to ensure that the state constraints at the start and end times are met.

In this paper, the cubic polynomial trajectory is used for intelligent vehicle trajectory planning. The parameterized curve of its trajectory with respect to time is defined as:

$$\begin{cases} x(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \\ y(t) = b_3t^3 + b_2t^2 + b_1t + b_0 \end{cases} \tag{1}$$

where $a_1, a_2, a_3$ and $b_1$ $b_2$ $b_3$ represent coefficients of the polynomial curve, and $x(t)$, $y(t)$ are positions of vehicle in global coordinate.

Equation (2) is an example of terminal and terminal constraints for lane change scenario of one-way two-lane roads, with constraints as follows. It is similar for other scenarios.

$$\begin{aligned} y(0) = 0, y(\tau) = W, \dot{y}(0) = 0, \dot{y}(\tau) = 0 \\ x(0) = 0, x(\tau) = D, \dot{x}(0) = u, \dot{x}(\tau) = u \end{aligned} \tag{2}$$

where $W$ represents the lane width, $\tau$ represents the total time for the vehicle to complete the planned trajectory, $L$ is the longitudinal length of the trajectory, and $u$ is the initial and final longitudinal velocity.

Substitute formula (1) into equation (2). Then the trajectory expression is determined as equation (3):

$$\begin{cases} x(t) = 2(u\tau - L)\left(\frac{t}{\tau}\right)^3 - 3(u\tau - L)\left(\frac{t}{\tau}\right)^2 \\ y(t) = -2W\left(\frac{t}{\tau}\right)^3 + 3W\left(\frac{t}{\tau}\right)^2 \end{cases} \tag{3}$$

The objective function of trajectory optimization is defined as follows:

$$\begin{aligned} J_{min} = \lambda_1 a^2 + \lambda_2 \tau^2 = \lambda_1 J_1 + \lambda_2 J_2 \\ a = max\left(\sqrt{\ddot{x}^2 + \ddot{y}^2}\right)\Big|_{t=0\sim\tau} \end{aligned} \tag{4}$$

where $a$ represents the maximum total acceleration of the vehicle, $\lambda_1$and$\lambda_2$ represent the weights of two variables. During the voyage, passenger comfort mainly depends on the size of vehicle acceleration. A greater acceleration will bring about a worse passenger experience [9, 10]. Therefore, $J_1$ in the objective function represents the optimal comfort performance of the desired trajectory during trajectory planning. $J_2$ is the efficiency of the planned trajectory, representing that the lane change operation is expected to be completed as soon as possible.

According to the above constraints and objective function, the trajectory interpolation curve is optimized to obtain the optimal trajectory as formula (5):

$$\begin{cases} x^*(t) = \dfrac{4.1271\frac{\lambda_2}{\lambda_1}^{1/6} \cdot W^{5/3}}{u^2} \cdot \left(\frac{t}{n}\right)^3 \\ \quad - \dfrac{6.2026\frac{\lambda_2}{\lambda_1}^{1/6} \cdot W^{5/3}}{u^2} \cdot \left(\frac{t}{n}\right)^2 + u \cdot t \\ y^*(t) = -2W \cdot \left(\frac{t}{n}\right)^3 + 3W \cdot \left(\frac{t}{n}\right)^2 \\ n = \dfrac{2.06W^{1.5}}{u^2}(kW)^{1/6} + 2.01\sqrt{W}(kW)^{-1/6} \end{cases} \tag{5}$$

where $x^*(t)$ and $y^*(t)$ represent the optimal trajectory with weights of $\lambda_1$ and $\lambda_2$, which need to be adjusted according to the information of the surrounding environment of the vehicle, to realize the generation of the optimal trajectory according to the current vehicle condition. With initial value $\lambda_1 = \lambda_2 = 0.5$, the trajectory is shown as Fig. 1, which will be further used as a reference trajectory.

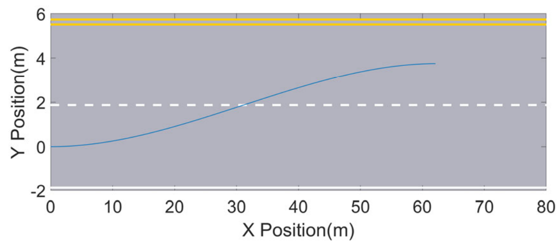

Fig. 1: Reference trajectory.

## 3. Trajectory Clustering

In trajectory planning, the intelligent vehicle needs to generate a group of alternative trajectories to get a new optimal trajectory when the current trajectory is not feasible [11-13]. Longitudinally expands the trajectory by changing the weight of the objective function, and laterally expands the trajectory by expanding the expected state at the end moment, generating the planned trajectory cluster.

### 3.1 Longitudinal Trajectory Clustering

The function of longitudinal expansion is to adjust the weight of comfort and efficiency, using the method of adjusting both the numerator and the denominator to make sure that the trajectory is evenly distributed in space, as follows:

$$\begin{cases} x^*(t) = \dfrac{4.1271p^{1/6} \cdot W^{5/3}}{u^2} \cdot \left(\frac{t}{n}\right)^3 \\ \quad - \dfrac{6.2026p^{1/6} \cdot W^{5/3}}{u^2} \cdot \left(\frac{t}{n}\right)^2 + u \cdot t \\ y^*(t) = -2W \cdot \left(\frac{t}{n}\right)^3 + 3W \cdot \left(\frac{t}{n}\right)^2 \\ n = \dfrac{2.06W^{1.5}}{u^2}(kW)^{1/6} + 2.01\sqrt{W}(kW)^{-1/6} \\ p = \dfrac{\lambda_2}{\lambda_1} = \dfrac{0.5 \pm j * d_k}{0.5 \mp j * d_k}, j = 0,1,\dots,N_X \end{cases} \tag{6}$$

where $p$ represents the ratio of the weights of the objective function, which is defined as the longitudinal turning factor, $d_k$ and $N_X$ represent the turning quantity and amplitude. The number of trajectories $N$ formed by expansion is as formula (7):

$$N = (2N_X + 1) \quad (7)$$

When the curvature of the trajectory is small enough, it is approximately equal to the second derivative of the longitudinal displacement with respect to the transverse displacement, which can be obtained as formula (8):

$$k \approx \frac{3W}{2K^2u^2} \quad (8)$$

where $k$ represents the maximum curvature of the trajectories. Since the purpose of the above calculation is to determine the expansion of the trajectory cluster, the error of the above approximate calculation is within the acceptable range. The maximum curvature constraint by the vehicle's bicycle model regulates the upper limit of factor $K$ as formula (9):

$$K < \sqrt{\frac{3W}{2k_{max}u^2}} \quad (9)$$

where $k_{max}$ represents the maximum curvature of the trajectory, which is later determined from the vehicle's bicycle model.

The lower limit of regulating factor K is constrained by the longest lane change distance as formula (10):

$$L \approx 2.4\sqrt{W}(2.88KW)^{-\frac{1}{6}}$$
$$K > \frac{1}{2.88W}\left(\frac{L_{max}}{2.4\sqrt{W}}\right)^{-6} \quad (10)$$

where $L_{max}$ represents the maximum longitudinal length of the trajectory.

Therefore, as formula (11), the turning factor $K$ satisfies:

$$\left(\frac{2.4\sqrt{W}}{L_{max}}\right)^{-6}\frac{1}{2.88W} < K < \sqrt{\frac{3W}{2k_{max}u^2}} \quad (11)$$

By considering the computational efficiency, the constraints of the turning factor can be rewritten as formula (12):

$$max\left(\frac{1}{9}, \frac{1}{2.88W}\left(\frac{L_{max}}{2.4\sqrt{W}}\right)^{-6}\right) < K < min\left(9, \sqrt{\frac{3W}{2k_{max}u^2}}\right) \quad (12)$$

The specific value of longitudinal extension $N_X$ can be obtained by combining the relationship between the regulating factor $K$ and the transverse extension $N_X$. Fig. 2 shows the trajectory cluster generated by longitudinal clustering of the reference trajectory.

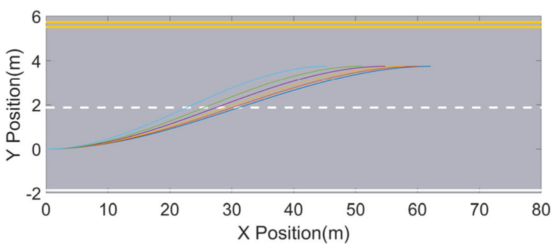

Fig. 2: Longitudinal trajectory clustering.

### 3.2 Lateral Trajectory Clustering

Lateral clustering of trajectory has been widely used since the early autonomous driving experiments, mainly to ensure to achieve a collision-free trajectory [3, 4]. Therefore, the expected state of the terminal moment is expanded as formula (13):

$$y(\tau) = W \pm i * d_W, i = 0,1, \ldots, N_Y \quad (13)$$

where $d_W$ and $N_Y$ represent the turning quantity and amplitude.

In the scenario of single two-lane lane change, only one extra terminal state on both sides is needed, and the displace width is half the width of the car (0.9m), which can produce collision-free trajectories. So $d_W = 0.9, N_Y = 1$ . The quantity of trajectories formed by expansion is as formula (15):

$$N = (2N_X + 1) * (2N_Y + 1) = 3 * (2N_X + 1) \quad (15)$$

For the trajectories that does not meet the original expected lane centerline, the subsequent planning process can be repeated to generate the new trajectory and achieve the target lane, as shown in Fig. 3.

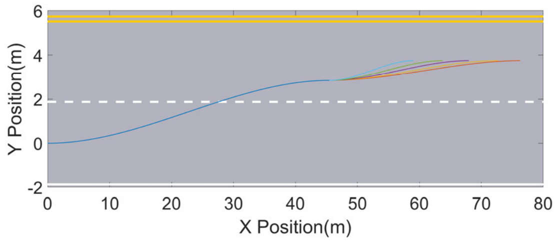

Fig. 3: Subsequent planning.

## 4. Optimal Trajectory Selection

The trajectory cluster generated by trajectory clustering is shown in the Fig. 4. The generated trajectory cluster can be divided into three sub-clusters: outer cluster, middle cluster and inner cluster, since the lateral expansion is once extended to both sides. The priority of each trajectory is defined and the prior trajectory is selected according to the surrounding environment information.

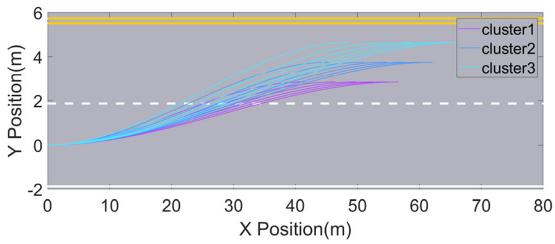

Fig. 4: Trajectory clustering classification.

Firstly, all the trajectories in the generated trajectory clusters will be checked for collision, and the detection algorithm is presented in the following subsection.

### 4.1 Collision Detecting Algorithm

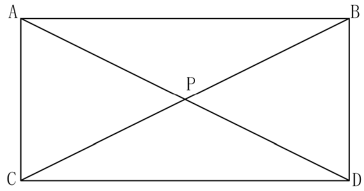

Fig. 5: Directionality of the cross product.

For two dimensional vectors $a = \begin{bmatrix} x_1 \\ y_1 \\ 0 \end{bmatrix}$ and $b = \begin{bmatrix} x_2 \\ y_2 \\ 0 \end{bmatrix}$, as formula (16), their cross product is:

$$a \times b = \begin{bmatrix} 0 \\ 0 \\ x_1 y_2 - x_2 y_1 \end{bmatrix} \quad (16)$$

Which can be denoted as formula (17):

$$a \times b = x_1 y_2 - x_2 y_1 \quad (17)$$

According to the directivity of the cross product, we can check whether a point is in a rectangular region as formula (18):

$$\begin{cases} (AB \times AP)(DC \times DP) > 0 \\ (CA \times CP)(BD \times BP) > 0 \end{cases} \quad (18)$$

Assuming that the trajectories of obstacle vehicles in the future has been obtained, we can judge whether a collision will occur in the future by whether some detecting points of the obstacle vehicles are within the boundaries of ego vehicle, as shown in Fig. 6 below.

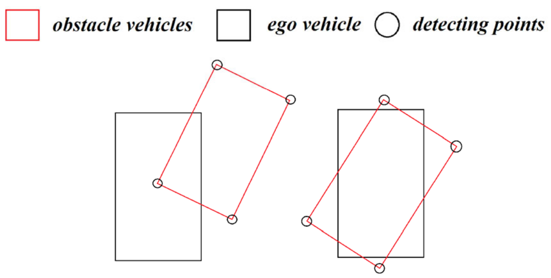

Fig. 6: Trajectory clustering classification.

In this method, the outline of the vehicle as a rectangle, select five detecting points in the obstacle vehicles, which contain four corners of the boundaries and geometrical center. If at any time in the future, one of these detecting points is located within the boundaries of ego vehicle, the collision is detected.

For the ego vehicle and obstacle vehicles, the trajectory of the four corners of its boundaries is calculated as follows:

$$\begin{bmatrix} x_A(t) & y_A(t) \\ x_B(t) & y_B(t) \\ x_C(t) & y_C(t) \\ x_D(t) & y_D(t) \end{bmatrix} = \frac{1}{2} B \times R + \begin{bmatrix} x(t) & y(t) \\ x(t) & y(t) \\ x(t) & y(t) \\ x(t) & y(t) \end{bmatrix} \quad (19)$$

where $B$ is a matrix related to the vehicle size, and $R$ is the rotation matrix as formula (20):

$$B = \begin{bmatrix} -l & -v & 0 & 0 \\ l & -v & 0 & 0 \\ l & v & 0 & 0 \\ -l & v & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} cos\ \varphi(t) & sin\ \varphi(t) \\ -sin\ \varphi(t) & cos\ \varphi(t) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (20)$$

where $l$ and $w$ is the length and width vehicles, $\varphi(t)$ represents the heading angle of the vehicles.

## 4.2 Priority of Trajectory

If all trajectories of the middle cluster are collision-free, all feasible trajectories of the vector to the target state are safe. Thus, the priority is defined as formula (21):

$$C = \lambda_1(1 - i) \quad (21)$$

which means that the priority of the middle cluster is higher than that of the outer cluster and inner cluster, and the higher the trajectory comfort is, the higher the priority is.

If there is collision in the middle cluster trajectory, there are obstacles in the area formed by the feasible trajectories. Thus, the priority is defined as formula (22):

$$C = d(1 - 0.1 * i) \quad (22)$$

where $d$ is the minimum dynamic distance between the trajectory and all obstacles, that is, the priority of the middle cluster is slightly higher than that of the outer cluster and inner cluster, and the higher the security, the higher the priority is. The tractor with the highest priority is called prior trajectory.

## 4.3 Model Predictive Analysis of Trajectory

The constraint conditions of the planned trajectory cluster at the initial and final moments include keeping the vehicle speed unchanged before and after the planning. By analyzing the transverse and longitudinal velocity of each trajectory in the trajectory cluster, it can be concluded that the longitudinal velocity has a slight change, while the lateral velocity has a significant change to complete the lane-changing behavior (see Fig. 7 and Fig. 8).

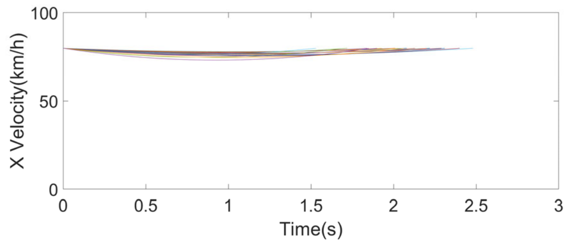

Fig. 7: Longitudinal velocity of trajectory cluster.

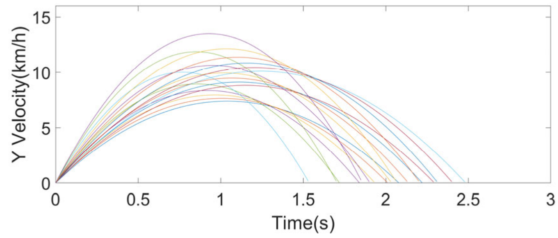

Fig. 8: Lateral velocity of trajectory cluster.

Therefore, state space expression of vehicle dynamics [14] is established as formula (23):

$$\begin{gathered} \frac{d}{dt} X = AX + B\delta, X = \begin{bmatrix} y \\ \dot{y} \\ \varphi \\ \dot{\varphi} \end{bmatrix} \\ Y = CX \end{gathered} \quad (23)$$

The parameter of the state space expression is calculated as formula (24):

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f} + 2C_{\alpha r}}{mu} & 0 & -u - \frac{2C_{\alpha f}\ell_f - 2C_{\alpha r}\ell_r}{mu} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2\ell_f C_{\alpha f} - 2\ell_r C_{\alpha r}}{I_z u} & 0 & -\frac{2{\ell_f}^2 C_{\alpha f} + 2{\ell_r}^2 C_{\alpha r}}{I_z u} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2\ell_f C_{\alpha f}}{I_z} \end{bmatrix} C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad (24)$$

where $\ell_f$ and $\ell_r$ represent the distances from the center of gravity to the front and rear axles, respectively; $I_z$ represents the moment of inertia of the vehicle about the vertical axis;

$C_{\alpha f}$ and $C_{\alpha r}$ represent the cornering stiffness of each front tire.

Based on the model above, the optimal control signal is solved by model predictive method. Under the condition of satisfying constraints of the vehicle, the planned trajectory is tracked, and feasibility of the current optimal trajectory is under the constraints is judged by the tracking effect.

When performing control, the predicted trajectory of the vehicle should, as far as possible, track the given prior trajectory. If the input $U(k)$ of the system is known, the future output $Y(k+1 \mid k)$ of the system can be predicted by solving the equation of state expression:

$$
\begin{gathered}
U(k)=\begin{bmatrix}\delta(k)\\ \delta(k+1)\\ \vdots\\ \delta(k+m-1)\end{bmatrix}\\
Y(k+1\mid k)=\begin{bmatrix}y(k+1\mid k)\\ y(k+2\mid k)\\ \vdots\\ y(k+p\mid k)\end{bmatrix}\\
Y(k+1\mid k)=S_xX(k)+S_uU(k)
\end{gathered}
\tag{25}
$$

The parameter of the predictive equation is calculated as formula (24):

$$
\begin{gathered}
S_x=\begin{bmatrix}C_dA_d & C_dA_d^2 & \cdots & C_dA_d^p\end{bmatrix}^T\\
S_u=\begin{bmatrix}C_dB_d & 0 & \cdots & 0\\ C_dA_dB_d & C_dB_d & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ C_dA^{p-1}B_d & C_dA^{p-2}B_d & \cdots & \sum_{i=1}^{p-m+1} C_dA_d^{i-1}B_d\end{bmatrix}
\end{gathered}
\tag{26}
$$

where $A_d=e^{AT_s}, B_d=\int_0^{T_s} e^{A\tau}d\tau \cdot B$, and $C_d=C$.

When tracking the trajectory, the tracking error should be minimized and the change of control action should be as small as possible. Therefore, the objective function is defined as follows:

$$
\begin{aligned}
J=&\sum_{i=1}^{p}\left\|\Gamma_i^y\big(y(k+i\mid k)-y(k+i)\big)\right\|^2\\
&+\sum_{i=1}^{m}\left\|\Gamma_i^u\Delta\delta(k+i-1)\right\|^2
\end{aligned}
\tag{27}
$$

where $\Gamma_i^y$ and $\Gamma_i^u$ is weight of each part of the objective function.

For vehicles, not only the above control objectives should be satisfied as much as possible through optimization, but also the following inequations as formula (28) should be satisfied to make the vehicle meet the vehicle steering system constraints:

$$
\begin{gathered}
\delta_{min}(k+i)\le\delta(k+i)\le\delta_{max}(k+i)\\
\Delta\delta_{min}(k+i)\le\Delta\delta(k+i)\le\Delta\delta_{max}(k+i)\\
i=0,1,2\cdots,m-1
\end{gathered}
\tag{28}
$$

where $\delta_{max}(k+i)$ and $\delta_{max}(k+i)$ are the minimum and maximum allowed steering wheel angles, and $\Delta\delta_{max}(k+i)$ and $\Delta\delta_{max}(k+i)$ are the minimum and maximum allowed steering speed.

In this case, the optimization problem is difficult be solved analytically, so it needs to be solved by numerical optimization method. As formula (29), he general form of quadratic programming problem is:

$$
\begin{gathered}
\min_x x^THx-g^Tx\\
\text{s.t. } Cx\ge b
\end{gathered}
\tag{29}
$$

where $H$ is Hessian matrix and $g$ is the gradient of objective function. The constrained optimization problem is described as a quadratic programming problem.

Substitute the prediction output into the objective function to obtain formula (30):

$$
J=(\Delta U(k))^TH\Delta U(k)-G(k+1)\Delta U(k) \tag{30}
$$

The Hessian matrix $H$ and $G(k+1)$is obtained as formula (31):

$$
\begin{gathered}
H=S_u^T(\Gamma^y)^T\Gamma^yS_u+(\Gamma^u)^T\Gamma^u\\
G(k+1)=2S_u^T(\Gamma^y)^T\Gamma^y(R(k+1)-S_x\Delta X(k))
\end{gathered}
\tag{31}
$$

Constraints on steering wheel Angle and steering wheel speed can be translated into:

$$
\begin{bmatrix}I_{m\times m}\\ -I_{m\times m}\\ L_{m\times m}\\ -L_{m\times m}\end{bmatrix}\Delta U<
\begin{bmatrix}-\Delta\delta_{max}(k)\\ \vdots\\ -\Delta\delta_{max}(k+m-1)\\ -\Delta\delta_{min}(k)\\ \vdots\\ -\Delta\delta_{min}(k+m-1)\\ \delta(k-1)-\delta_{max}(k)\\ \vdots\\ \delta(k-1)-\delta_{max}(k+m-1)\\ \delta_{min}(k)-\delta(k-1)\\ \vdots\\ \delta_{min}(k+m-1)-\delta(k-1)\end{bmatrix}
\tag{32}
$$

which is denoted as

$$
C\Delta U(k)\ge b(k+1) \tag{33}
$$

where $I$ and $L$ represent identity matrix and lower triangular matrix

The constrained optimization problem is transformed into the following quadratic programming problem:

$$
\begin{gathered}
\min_{\Delta U_m(k)}(\Delta U(k))^TH\Delta U(k)-G(k+1)\Delta U(k)\\
\text{s.t. } C\Delta U(k)\ge b(k+1)
\end{gathered}
\tag{34}
$$

Solving this quadratic programming problem can predict the tracking effect of vehicles on the prior trajectory, as shown in the Fig. 9.

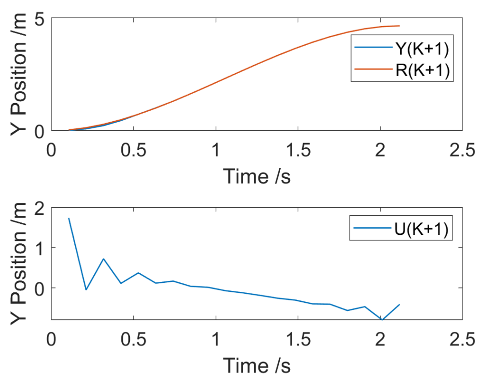


Fig. 9: prior trajectory tracking.

In order to evaluate the prediction and tracking effect of the prior trajectory under the current constraints, the qualification function is defined as formula (35):

$$
Q=max\|Y(k+1)-R(k+1)\| \tag{35}
$$

where $Q$ represents the maximum displace during tracking.

When $Q<0.1$, it indicates that the trajectory with the highest priority conforms to the vehicle dynamics constraints, and this trajectory is regarded as the optimal trajectory. When $Q>0.1$, test downward according to decreasing priority until the trajectory with $Q<0.1$ is regarded as the optimal trajectory.

## 5. Simulations and Discussions

Scenario 1: There is an obstacle vehicle far ahead in the current lane, and the speed is lower than the ego vehicle. No vehicles in the target lane.

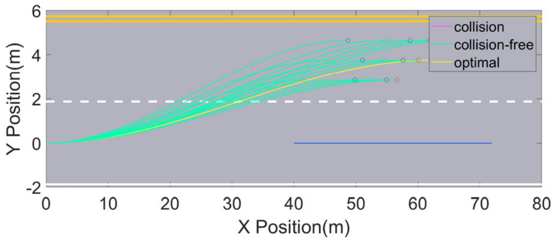

Fig. 10: Trajectory planning in Scenario 1.

Scenario 2: There is an obstacle vehicle ahead in the current lane, and the speed is lower than the ego vehicle. No vehicles in the target lane.

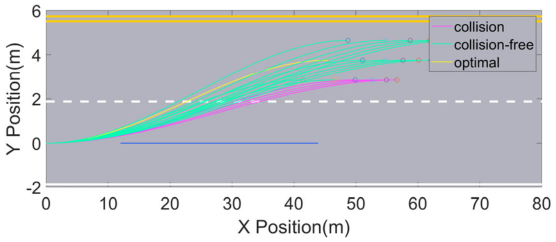

Fig. 11: Trajectory planning in Scenario 2.

Scenario 3: There is an obstacle vehicle ahead in the current lane, and the speed is lower than the ego vehicle. Another obstacle vehicle is in the target lane.

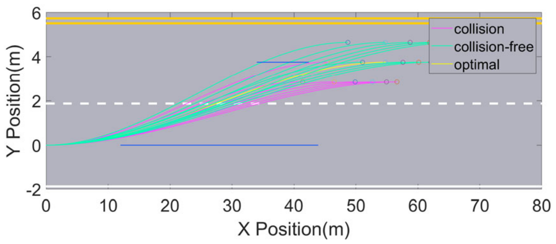

Fig. 12: Trajectory planning in Scenario 3.

The collision-free trajectory cluster and prior trajectory of each scenario is shown in Fig.10, Fig.11, and Fig.12. After tracking with optimal control signal, the prior trajectory in each scenario satisfies $Q < 0.1$ and can be taken as optimal trajectory.

## 6. Conclusion

This paper presented a hybrid trajectory planning strategy with interpolation curve optimization method under the constraints of collision avoidance and vehicle dynamics. The cubic polynomial curve was firstly optimized to generate a reference trajectory. Then, in order to generate collision-free trajectory, the reference trajectory was clustered. Furthermore, an algorithm was designed to detect the collision of the trajectories. The priority of the trajectories was defined to determine the prior trajectory. The prior trajectory was tracked with optimal control signal and judged by the tracking effect. Simulation results showed that the trajectory planning strategy is suitable in different scenarios. Future works will focus on extending the proposed strategy to more types of roads and scenarios.

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